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Enhancing Mathematical Reasoning in Primary School With the Strategic Board Game Othello

Erik Ottar Jensen[1], Emilie Madeline Hersaa Nehammer[2] and Anna Louise Eriksen[3]
[1]Aalborg University, Copenhagen, Denmark
[2]Virum Skole, Lyngby-Taarbæk, Denmark
[3]Lindevangskolen, Frederiksberg, Denmark

erikoj@ikp.aau.dk
nehammer@hotmail.com
annaeriksen93@live.dk

Abstract: This study explores how the board game Othello can enhance primary school students' mathematical reasoning. Mathematical reasoning is increasingly emphasized in international mathematics curricula, yet both teachers and students face challenges in teaching and learning this important skill. Research shows that strategic games, like Othello, can develop thinking abilities related to mathematical reasoning by providing a context for students to engage in reflective thinking, anticipate future moves, and develop reasoning strategies. However, there is a need to understand how teachers can effectively utilize these games in the classroom to foster mathematical reasoning. We address this, through a design-based research approach consisting of a hermeneutic literature study and the development of design principles for a teaching intervention in a fifth-grade classroom. Data from the intervention is collected through participant observations and group interviews. The findings suggest that Othello can serve as a context for students to engage in mathematical reasoning by making and justifying claims and presenting logical arguments. The study proposes three design principles to scaffold mathematical reasoning during Othello gameplay. These principles focus on introducing and reinforcing the use of the "if...then" formulation, promoting exploratory talk, encouraging reflection on strategies, and fostering collaborative reasoning. The results indicate that the design principles positively impacted students' ability to reason mathematically. This paper contributes to the field of mathematics education and game-based learning by providing a practice-oriented perspective on designing mathematical instruction for reasoning using a specific board game in a primary school setting. The findings offer insights into the potential of strategic board games like Othello to enhance students' mathematical reasoning skills. The design principles proposed in this study can guide teachers in developing effective instructional approaches to support students' mathematical reasoning development.

Keywords: Mathematics education, Mathematical reasoning, Exploratory talk, Game-based learning, Strategic board games, Primary school students

1. Introduction

The study aims to understand how the board game Othello can be used to design teaching interventions developing primary school students' mathematical reasoning. We define mathematical reasoning as being able to make a claim of a mathematical nature and to argue why this claim is true by explaining and justifying one's ideas. This definition is inspired by McFeetors and Palfy (2018) and Niss and Jensen (2002) and will be elaborated in section 3.1. Mathematical reasoning is considered a central part of doing mathematics and has recently gained a prominent role in international mathematics curricula and mathematics education research (Jensen & Skott, 2022). However, mathematics teachers find it challenging to teach mathematical reasoning and students find it difficult to learn (G. J. Stylianides et al, 2017). Strategic and logic games have been shown to develop aspects of thinking related to mathematical reasoning (McFeetors & Palfy, 2018) like generalizing and reasoning for a winning strategy (Day 2014). Strategic board games can potentially contribute to the development of students' mathematical reasoning by providing a context for reasoning in the development and testing of hypotheses as they encourage students to anticipate future moves and reflect on game situations (Olson, 2007). But there is still few intervention studies aiming to support teachers in designing learning experiences with mathematical reasoning in primary school (G. J. Stylianides et al, 2017). Through a design experiment (Cobb et al, 2003) with the board game Othello and a practice-oriented perspective on how primary school teachers can design mathematical instruction for reasoning, this study addresses this research gap. We do this by posing the research question: What design principles are relevant for planning lessons promoting students' mathematical reasoning with Othello?

2. Research Design

Our methodological research approach is Design-Based research (DBR) (Barab & Squire, 2004) which is well suited for developing new forms of instruction, enabling researchers to iteratively design, refine, and test interventions in real-world settings (Cobb et al, 2003). This approach considers the complex interactions between learners, the instructional environment, and the content being learned. The iterative process allows researchers to refine their understanding of student learning and identify design principles that inform the development of new instructional approaches (Cobb et al, 2003).

This study is the first iteration with the aim to create sound design principles for further testing and so the validity of the principles is open to critique based on the lack of iterations. Our research design consists of three parts. First, a hermeneutic literature study (Boell & Cecez-Kecmanovic, 2014) of mathematical reasoning and games, supplemented by identifying Othello's potential for reasoning which provides the theoretical frames for the study. Secondly, a design of a teaching intervention generating guiding design principles (Hanghøj et al. 2022). Thirdly, an intervention in a fifth-grade math class (approx. 12-year-olds) to investigate and refine the design principles. The intervention spanned three 120-minute lessons. The class was recruited by our collaborator who is the chairman of the Danish Othello Association. The intervention teacher responsible for conducting the lessons, was Anna Louise Eriksen, co-author of this paper.

2.1 Design Principles

As proposed by Hanghøj et al. (2022) we created design principles to articulate our theoretical assumptions into specific guidelines for scaffolding mathematical reasoning in the intervention. This was guided by results from the hermeneutic literature study and the analysis of Othello (see section 3). The design principles are intended to be used when designing mathematics lessons aimed at using Othello to develop primary school students' mathematical reasoning. They are aimed at engaging students in 'exploratory talk' (Mercer et al, 1999) as a means of expressing mathematical reasoning. To benefit the most from the design principles the lessons should be structured as the three-phased model for inquiry-based mathematics teaching (Blomhøj et al, 2022): 1) Setting the scene for the students' activities. In the intervention the primary component in this phase, is an initial didactic section where the teacher establishes the context for the following adidactic part (Brousseau, 1997). 2) Allow for sufficient time and freedom for students' independent inquiry-oriented activities. Here the students engage in the Othello activity. 3) Share reflections, experiences, and learning. In this phase the teacher refines and reinforces the understanding of the mathematical reasoning that emerged during the Othello activity by promoting a reasoning-focused classroom environment and encouraging students to share their strategies and thinking through inquiries about their thought processes (Whitenacke & Yackel, 2002).

To gather data, two of the authors performed participant observations (Creswell, 2014) during all three lessons, documenting observations in a detailed logbook and collected the materials produced by students. The observations were directed towards two pairs of students in the class and the teacher. Following the intervention, we conducted two semi-structured group interviews (Brinkmann & Kvale, 2015) with the two pairs of students. We recorded and transcribed the students' group work, the discussions in the classroom initiated by the teacher and the interviews. The collected data (logbooks, transcriptions, interviews, and student-produced materials) was examined in relation to the design principles and findings from the literature study by asking the following questions: Which aspects of mathematical reasoning were promoted in the intervention? How did the design principles support this and what were the challenges? With these questions, we aimed at understanding the three design principles potential for engaging students in exploratory talk when using Othello in a mathematics teaching practice.

3. Theoretical Frame

The theoretical frame for our investigation and understand mathematical reasoning in analog games was developed through a hermeneutic literature study (Boell & Cecez-Kecmanovic, 2014). This was done through a cyclical and iterative approach consisting of searching for literature, sorting, selecting, acquiring, reading, identifying, refining, searching, ect. In this process parts of the literature are continuously understood and related to the overall understanding of the phenomenon (Boell & Cecez-Kecmanovic, 2014). Our intention was to get a deep and broad understanding by continuously and consistently interpreting the literature in relation to other relevant literature. We used two overarching searches, one trying to understand mathematics reasoning in general and one related to analogue games. The searchers were a combination of searching for keywords in the Aarhus University search engine combined with citation searching (Boell & Cecez-Kecmanovic, 2014), in the curriculum for the master's in mathematics education in Denmark and articles found in a prior search for a review of digital games and mathematical reasoning (Jensen & Skott, 2022). See Eriksen and Nehammer (2022) for further details. The following two sections presents our results from the review that also act as our theoretical frame for understanding mathematical reasoning in analogue games.

3.1 Mathematical Reasoning

The literature study of mathematical reasoning found that different definitions of reasoning emphasize either structural or processual aspects (Jeannotte & Kieran, 2017). Structural aspects understood as; 'A good' or 'a logically valid argument' (Lindhart et al, 2010), 'a chain of both formal and informal arguments' (Niss & Jensen, 2002) 'a reasoning that involves patterns' (delMas, 2004). The processual aspect, highlight that mathematical reasoning involves communication with oneself or with others in either writing or speech and is understood as; Definitions that emphasize processual structure include descriptions such as 'to justify', 'to modify', and 'to evaluate' (McFeetors & Palfy, 2018; Whitenacke & Yackel, 2002). Further we found arguments that that reasoning for arguments should be understood relative to the age and mathematical ability of the students (Lindhart et al, 2010). In agreement with the literature study, we define mathematical reasoning as being able to make a claim of mathematical nature and to argue why this claim is true by explaining and justifying one's ideas. This includes following others' arguments and being willing to reformulate one's own arguments if they turn out to be untrue because of another argument presented (McFeetors & Palfy, 2018; Niss & Jensen, 2002). To design for mathematical reasoning in class we aim to facilitate 'exploratory talk' (Mercer et al, 1999), a form of communication similar to our definition of mathematical reasoning, as it is characterized by students explaining and justifying their reasoning in a joint discussion, listening to each other, and positively accepting challenges from both the teacher and other students.

3.2 Development of Mathematical Reasoning in Othello

Othello is a competitive two-player board game played on an 8x8 board with identical game pieces with one white and one black side. Players compete to have most pieces of their color, either black or white facing up at the end of the game. Players take turns placing one piece on the board and flipping over any of the opponent's pieces that are sandwiched between her own pieces. There are no chance factors in the game and all information is available to both players making it a combinatorial strategy game (Fraenkel, 2002) a category of games that have potentials in developing students' mathematical reasoning (Day, 2014; Houssart & Sams, 2008). Specifically, Othello offers an opportunity for players to gain experience and recognize patterns in the game by understanding the relationship between the two variables, black and white. By combining our definition of mathematical reasoning with the understanding of Othello as a combinatorial strategy game we argue that the game can be used as a context for students to devise and present mathematical reasoning in the form of 'a good argument,' 'a logically valid argument,' or 'a chain of informal arguments' for a claim of mathematical nature (which relate to the structural aspects) and, by 'explaining' and 'justifying' their ideas (which relate to the processual aspects). In our intervention design we assume that there will be a progress towards refining claims through arguments. To assess this, we created a table that indicates progress in student reasoning and an envisioned example with Othello (see Figure 1). It is partly based on the taxonomy presented by Díez-Palomar & Olivé (2015) and draws a parallel between 'exploratory talk' and 'Interaction Type 3: Dialogic Interaction', that is characterized by students engaging in the interaction and establishing dialogues based on validity claims expressed in the dialogue.

Interaction type	Claim characteristics	Reasoning	Envisioned example
1: Exchange of information.	Claims without arguing by explaining and justifying.	No reasoning.	'It is disadvantageous to start.'
2: Non-dialogic.	Invalid arguments based on everyday experiences to explain and justify a claim.	Everyday reasoning.	'I win at Othello because I often play board games.'
	Invalid arguments based on an asymmetric power position to explain and justify a claim.	Reasoning based on an asymmetric power position.	'If I place my color in the corners of the game board, I will win because my teacher said so.'
3: Dialogic interaction.	Correct and meaningful arguments for a claim by explaining and justifying.	Correct and meaningful reasoning.	A good or logically valid argument: 'If I place my piece in this square, I can flip three pieces because I am trapping my opponent's pieces horizontally.'

Interaction type	Claim characteristics	Reasoning	Envisioned example
			A chain of informal arguments 'If I place pieces in the corner, my opponent cannot flip them, and then I have the opportunity to flip my opponent's pieces both diagonally and along the edges'

Figure 1: The Interaction types and associated types of mathematical reasoning, or lack thereof, that we expected students to be able to express in a game situation

4. Results

To improve clarity for the reader, this section is structured so that each design principle is presented first through our a priori intention for the principle and secondly through the analysis of the empirical investigation.

4.1 Principle one: Introduce and continually Repeat the 'if...then' Formulation

Intention: By presenting the 'if…then' formulation in the introduction and continually repeating this in the interaction with the students, the teacher can explicitly encourage 'exploratory talk'. The formulation can be used to express different degrees of reasoning in student dialogue ranging from mathematical reasoning to 'good' or 'logically valid argument' or a 'chain of informal arguments' for a claim by 'explaining' and 'justifying' their ideas (Mason, 2002). This can in different ways express that students are on the way to develop an advantageous strategy (Houssart & Sams, 2008).

Analysis: In the intervention, the teacher was explicit about the 'if...then' formulation in all introductory presentations for the double lessons. In the first lesson, this occurred after the students engaged in one-on-one play to internalize the rules, which is crucial if the game's potential for developing students' mathematical reasoning abilities is to be realized (McFeetors & Palfy, 2018). In the first two lessons of the intervention, several students expressed themselves in a way that was consistent with the following statement:

> *Student: 'If we place a piece there, then we can flip these three.'*

Here using the 'if...then' formulation, the students were able to explain and justify a claim with a simple but logically valid argument that considered the premises constituted by the rules of the Othello game. The simple arguments based on validity claims, which the students presented using the 'if...then' formulation, indicate an early use of ‘Interaction Type 3’ (Díez-Palomar & Olivé, 2015). In the second round of play, groups took longer than the first as: 'Several students take time to consider where to place their pieces while also using 'if...then' more in their conversation with their partner.' (logbook). Especially in the second and third double lesson, we find examples of students attempting to formulate a more long-term strategy by using the 'if...then' formulation as a basis for predicting up to three consecutive moves, which could be considered a progression in their reasoning. At this stage in the process, students were able to formulate chains of informal reasoning as opposed to the beginning of the intervention where they mainly presented simple but logically valid arguments. The following statement is an example of mathematical reasoning consisting of 'a chain of informal arguments' for a claim about the consequences of the game pair's choice of moves for the game's development and the opponents' opportunities:

> *Student: '...if we had placed it there, then our white piece would have disappeared. Then you the opponent) could have taken our white piece, and we couldn't have placed it there.'*

The active use of the 'if...then' formulations in the Othello activities indicate that ‘design principle one’ had a positive effect on developing students' mathematical reasoning through Othello. When the student pairs internally discuss their future moves during the Othello activity and using the 'if…then' formulations we saw several instances of ‘exploratory talk’ and what we will call a rich environment for developing mathematical reasoning. Further the use of the 'if…then' formulations were used to express increasingly advanced forms of reasoning through the intervention.

4.2 Principle Two: Allow one Pair to Compete Against Another Pair

Intention: Like most other classic and abstract board games, Othello is played in a one-on-one setup which actively discourages players from communicating about the moves they choose during the game. McFeetors and Palfy (2018) find that students have better opportunities to reason when they have both allies and opponents, and that students are more likely to both express and refine reasoning when they must justify a move to a teammate. Therefore, to promote 'exploratory talk,' we organize the students in groups of four, with one pair playing against another pair.

Analysis: We observed in the first double lesson that students were more likely to produce reasoning when playing one pair against another pair, rather than when playing one student against another student and that the students produced more correct and meaningful reasoning as the intervention progressed. An example of this was when a student in the first round playing one-on-one made a claim without explaining or justifying why the claim was true:

> *Student: 'It will ruin everything for you if I place it here,'*

In the second round, the same student made a claim where she now provided reasoning and explained why the claim was true:

> *Student: 'So I think we should block here because... if we put it there, then we turn here, but then they also have the opportunity to put theirs there, and then they also have all those.'*

We understand this as a form of explorative talk as the students here exchange an argument and explain and justify their reasoning while also listening to each other. During the intervention the students became more inclined to listen to and evaluate each other's reasoning and improve a move through investigative talk and exchange of reasoning. Because the students were put in a situation where they have to justify their mathematical thinking to someone else (their teammate), it creates good conditions for them to jointly create an even stronger argument or find another way to solve the given problem (Whitenacke & Yackel, 2002). The configuration with both teammates and opponents thus allowed students to come up with new moves that were more advantageous to them by evaluating each other's reasoning. However, Othello being a game and a social situation it also created conflicting and inconsistent findings in terms of the students' reasoning. One was that we did not observe that students used a strategy inspired by reasoning of the opposing pair. One student expressed: '...The others shouldn't hear our plans.'. In this sense it should be considered that the competitive element of Othello can create limitations for the students when expressing their reasoning to each other. Another finding was that the number of ‘unjustified and power claims’ (Díez-Palomar & Olivé, 2015) such as 'Black has a lot of advantages', did not change notably over the six rounds of play. Here is another example of how these claims were observed in the interaction.

> *Student1: 'We could put it there.'*
>
> *Student2: 'Yes, I think we should do that.'*

Examples of power claims are: 'Yes, we are so smart, folks.' Or 'That's why we're winning.' The reason students used 'unjustified claims' and 'power arguments', which respectively correspond to ‘Interaction type 1 and type 2’, might be associated with the tendency of primary school students to use inappropriate forms of conversation as suggested by Díez-Palomar and Olivé (2015) and Larsen and Lindhardt (2019).

4.3 Principle three: Have the Students Play Specific Variations of the Game

Intention: As an integral part of the Othello activity, the teacher (and later, the students) can continuously try to change the premises of the game. This can be done, for example, by adjusting starting positions and letting students try out different game situations the students must start their game from. In this way, students are motivated to reason in new ways, which can contribute to their ability not only to apply specific strategies in specific situations, but also to understand why those strategies work (Houssart & Sams, 2008). This in turn requires the use of the 'if...then' formulations in new contexts, which further promotes their ‘exploratory talk’, if challenges are received positively (Mercer et al, 1999).

Analysis: By combining input from our contact from the Danish Othello Association, four young Othello talents and the rule book from Othello we chose four strategies as the basis for development of seven student worksheets. Each of these changed the game's premise offering different starting positions that we envisioned would encourage students to reason mathematically, Guiding questions and some of the worksheets formulated that the students should use 'if... then' in their response to the task. The worksheets were introduced from the second lesson. We observed how students started to plan more than one move ahead and predict the opponents' future moves which is crucial for formulating a winning strategy (Houssart & Sams, 2008). One example of this is:

> *Student: 'We're White, right? We start by putting one here, then we take the two. Then they put a Black one there and take them.'*

The statement is an expression of a student considering not only how many pieces their move allows them to flip, but also how the move will affect their opponent's ability to flip pieces. This kind of reasoning was more frequent in the game rounds where the students played based on a worksheet.

5. Discussion

Inspired by McFeetors et al. (2018) we discuss three central arguments, each representing a distinct perspective on whether mathematical reasoning occurs when students explain and justify the claims they make in connection with the intervention.

Firstly, students used mathematical aspects in their considerations about the strategies they were introduced to in two different ways: By applying already acquired mathematical knowledge they have met earlier in school, and by using mathematical aspects they only occasionally had encountered in school previously thereby the students integrated mathematical aspects in both a retrospective and a forward-looking perspective. This included using knowledge of; diagonals and symmetry axes in connection with the worksheets, simple arithmetic by counting the number of pieces they could flip and, of the coordinate system when discussing the placement of pieces on the game board, as the playing board was divided vertically into the numbers 1-8 and horizontally into the letters a-h.

Secondly, if mathematical reasoning, among other things, involves generalization based on patterns (delMas, 2004), it can be argued that the strategies students develop during the intervention represent a form of generalization of the patterns they became familiar with in the game. Thus, we can claim that they are indeed engaged in a kind of mathematical reasoning.

Thirdly, we became aware of how reasoning mathematically about strategies in a board game is a communicative process. When students try to provide informal explanations and justifications for a claim in an engaging context that gives them a sense of continuous progression and coherence between their reasoning, it can be said that they are practicing the preliminary exercises for a more formal work with mathematical reasoning. For these three reasons, we argue that students articulated mathematical reasoning in their work with Othello.

Although we believe that students can present mathematical reasoning when playing Othello in an educational context where the three design principles are applied, we also acknowledge that our design has limitations. For instance, a student responded to something entirely different when the teacher asked a follow-up question meant to encourage reasoning, addressing whether the student had considered more than just her upcoming move:

> *Students: 'We used code words so the others wouldn't hear what we were thinking. Salad, relish, and stuff.'*

The response is aimed at concealing a strategy for the opponent which didn't encourage further mathematical discussion in the group. It indicates that the student did not understand that the aim of the lessons was to formulate correct and meaningful arguments for claims in the form of a reasoning. It could be because the students understand the activity as gameplay instead of mathematics (Jensen & Hanghøj, 2022). One issue with validating our results is the entanglement of the design principles when implemented (Hanghøj et al, 2022). It is difficult to pinpoint the exact impact of each principle when we successfully encourage the students to reason. The worksheets changed the game's premises through various setups but were also applied in pair vs. pair scenarios, even incorporating 'if... then' statements in some cases. This complexity makes it hard to assess each principle individually. As a result, we propose that it is the combined effect of the principles that contributes to the development of student reasoning.

6. Conclusion

In a DBR-approach we conducted a literature study of mathematical reasoning and the potential for analog games in developing this in primary school students. Using the potential of the board game Othello and the findings of the literature study we created three design principles. The principles are intended to be used by teachers to plan mathematics lesson aimed at promoting students' mathematical reasoning with Othello. We investigated the principles empirically through a teaching intervention in a fifth-grade math class. We found that students engaged in ‘exploratory talk’, which was demonstrated by the students presenting reasoning involving both structural and procedural aspects. The fact that students were able to engage in exploratory talk during the Othello activities indicates that the intervention had potential for developing students' mathematical reasoning.

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